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|  | **Level 2 Further Maths Overview** |
| Unit 1:Number and Algebra I | * Numbers and the number system
* Simplifying expressions
* Solving linear equations
* Manipulating Surds
 | * Algebra and number
* Expanding brackets
* **The binomial expansion**
* Product Rule For Counting
 |
| Unit 2: Algebra II | * Factorising
* Rearranging formulae
* Simplifying algebraic fractions
 | * Solving linear equations involving fractions
* Completing the square
 |
| Unit 3: Algebra III | * Function notation
* **Domain and range of a function**
* Composite functions
* Graphs of functions
* **Graphs of functions with up to three parts to their domains**
 | * Graphs of linear functions
* Finding the equation of a line
* Graphs of quadratic functions
* Inverse functions
* **Graphs of exponential functions**
 |
| Unit 4: Algebra IV | * Quadratic equations
* Simultaneous equations in two unknowns
* **The factor theorem**
* Algebraic proof
* Sequences
 | * Linear inequalities
* Quadratic inequalities
* Indices
* **Limiting value of a sequence**
* **Simultaneous equations in three unknowns**
 |
| Unit 5: Coordinate Geometry | * Parallel and perpendicular lines
* The distance between two points
* The midpoint of a line joining two points
* Dividing a line in a given ratio
 | * Equation of a straight line
* The intersection of two lines
* **Equation of a circle**
 |
| Unit 6: Geometry I | * Pythagoras’ theorem
* Angle facts
* Circle theorems
* Geometric proof
* Trigonometry in two dimensions
* Trigonometric functions for angles of any size
 | * The sine and cosine graphs
* The tangent graph
* **Solution of Trigonometric Equations**
* **Trigonometric identities**
 |
| Unit 7: Geomerty II | * The area of a triangle
* The sine rule
* The cosine rule
 | * Using the sine and cosine rules together
* Problems in three dimensions
* **Lines and planes in three dimensions**
 |
| Unit 8: Calculus | * **The gradient of a curve**
* **Differentiation**
* **Increasing and decreasing functions**
* **The second derivative**
 | * **Differentiation using standard results**
* **Tangents and normal**
* **Stationary points**
 |
| Unit 9: Matrices | * **Multiplying matrices**
* **Transformations**
* **The identity matrix**
 | * **Transformations of the unit square**
* **Combining transformations**
 |

# **Key Points For Teaching Further Maths**

* GCSE Maths content is signposted thoughout with the use of highlighting/coloured text to link with the 5-year-SOW. Further Maths only content is **highlighted in bold** on the overview.
* Objectives have been taken, in general, from the 5-year-SOW.
* Please focus, in particular, on topics exclusively found in Level 2 Further Maths. Although some topics do crossover with GCSE Maths/Level 2 Further Maths, please be aware that the level of difficulty on questions for these topics will generally be higher.
* List of Further Maths exclusive topics:

	+ **The binomial expansion**
	+ **Domain and range of a function**
	+ **Graphs of functions with up to three parts to their domains**
	+ **Graphs of exponential functions**
	+ **The factor theorem**
	+ **Limiting value of a sequence**
	+ **Simultaneous equations in three unknowns**
	+ **The intersection of two lines**
	+ **Equation of a circle (centre (a, b) )**
	+ **Trigonometric functions for angles of any size**
	+ **Solution of Trigonometric Equations**
	+ **Trigonometric identities**
	+ **Lines and planes in three dimensions**
	+ **The gradient of a curve**
	+ **Differentiation**
	+ **Increasing and decreasing functions**
	+ **The second derivative**
	+ **Differentiation using standard results**
	+ **Tangents and normal**
	+ **Stationary points**
	+ **Multiplying matrices**
	+ **Transformations**
	+ **The identity matrix**
	+ **Transformations of the unit square**
	+ **Combining transformations**
* There will be resources provided for the teaching of Level 2 Further Maths, including animated PowerPoints and worksheets. Exam materials can be found at *allaboutmaths* and good resources can be found on Dr Austin Maths/CorbettMaths. You can also set homework tasks on Sparx Maths and Dr Frost Learning.

# **Unit 1 – Number and Algebra I**

|  |  |
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|  | **Unit 1:****Number and Algebra I** |
| Unit 1:Number and Algebra I | * Numbers and the number system
* Simplifying expressions
* Solving linear equations
* Manipulating Surds
 | * Algebra and number
* Expanding brackets
* **The binomial expansion**
* Product Rule For Counting
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**Binomial Expansions**

* **Expand (*a* + *b*)*n* for positive integer *n***
* **Use Pascal’s triangle to work out the coefficient of *x*3 in the expansion of (3 + 2*x*)5**

*Topics that appear on Further Maths but crossover with GCSE Maths:*

Students should already be familiar with and fluent in the following topics. Opportunity to stretch and challenge these topics:

**Numbers and the number system**

* Understand and use the correct hierarchy of operations.
* Understand and use decimals, fractions and percentages.
* Understand rounding and give answers to an appropriate degree of accuracy.
* Understand and use ratio and proportion.
	+ An appreciation that $a\% of b$ is the same as $b\% of a$ can be useful.
	+ Students may find it useful to change ratio problems into fractional problems using $x:y=a:b ⇔ \frac{x}{y}=\frac{a}{b}$

**Algebra and Number**

* The basic processes of algebra
* Students should understand when and why algebraic expressions can be combined.

**Simplifying Expressions**

* Know the definition of a term and an equation
* Identify like and unlike terms
* Recognise that like terms can be combined together
* Simplify expressions by cancelling, e.g. = 2x
* Multiply together two simple algebraic expressions, e.g. 2a × 3b

**Solving Linear Equations**

* Understand algebraic representations a + 5, b – 4, 3c, d/3, e2 etc.
* Solve linear equations, with integer coefficients, in which the unknown appears on either side of the equation
* Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution
* Solve linear equations in one unknown, with integer or fractional coefficients
* Substitute into a formula, and solve the resulting equation
* Solve linear equations, with integer coefficients, in which the unknown appears on both sides of the equation
* Write expressions to solve problems
* Understand the ≠ symbol (not equal), e.g. 6x + 4 ≠ 3(x + 2)
* Form equations involving more complex shapes and solve these equations

**Manipulating Surds**

* Use surds with add and subtract
* Use surds with multiplication and division
* Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2
* Simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3)
* Recall and use Pythagoras’ Theorem in 2D, including leaving answers in surd form
* Expanding brackets containing surds including conjugate pairs for example:

($1- \sqrt{2})(1+ \sqrt{2}$)

* Rationalise the denominator involving basic surds for example: $√3$
* Rationalise the denominator involving surds of the form √5+1 with integer numerator
* **Rationalise the denominator involving surds as numerator and denominator**

**Expanding Brackets**

* Multiply a single term over a bracket
* Simplify expressions involving brackets, i.e. expand the brackets, then add/subtract
* Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to (2*x* + 3*y*)(3*x* – *y*)
* Know that squaring a linear expression is the same as expanding double brackets
* Define a ‘quadratic’ expression
* Expand the product of more than two linear expressions

**Product Rule for Counting**

* List all outcomes for single events, and combined events, systematically
* Use the product rule for counting (i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways)

# **Unit 2 – Algebra II**

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|  | **Unit 2:****Algebra II** |
| Unit 2:Algebra II | * Factorising
* Rearranging formulae
* Simplifying algebraic fractions
 | * Solving linear equations involving fractions
* Completing the square
 |

*Topics that appear on Further Maths but crossover with GCSE Maths:*

Students should already be familiar with and fluent in the following topics. Opportunity to stretch and challenge these topics. **Bold topics provide particular opportunity.**

**Factorising**

* Recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms
* Factorise algebraic expressions by taking out common factors
* Argue mathematically to show algebraic expressions are equivalent
* Factorise quadratic expressions of the form *ax*2 + *bx* + *c* when *a* = 1
* Factorise quadratic expressions using the difference of two squares when a=1
* Factorise quadratic expressions of the form *ax*2 + *bx* + *c* when *a* > 1
* Factorise quadratic expressions using the difference of two squares when a > 1
* Find the roots of a quadratic function algebraically and represent these on a graph

**Rearranging Formulae**

* Rearrange simple equations
* Change the subject of a simple formula, i.e. linear one-step, such as *x* = 4*y*
* Change the subject of a formula, including cases where the subject is on both sides of the original formula (not when factorising is required), or involving fractions and small powers of the subject
* Change the subject of a formula involving the use of square roots and squares

**Simplifying algebraic fractions**

* Simplify algebraic fractions
* Multiply and divide algebraic fractions
* Add and subtract simple algebraic fractions
* Solve quadratic equations arising from algebraic fraction equations

**Solving Linear Equations involving fractions**

* Solve linear equations with denominators or involving fractional coefficients before brackets

**Completing the Square**

* Rearrange quadratic into completed square form when a=1
* Find the vertex of a quadratic function algebraically and represent this on a graph
* Rearrange quadratic into completed square form when a>1
* Solve quadratics by completing the square where a>1
* Find the vertex of a quadratic function algebraically and represent this on a graph
* **Students should be able to deal with expressions with a coefficient of** $x^{2}$ **other than 1.**
* **The most able students will appreciate that** $c$ **is the greatest (or least) value of an expression of the form** $a(x+b)^{2}+c$

# **Unit 3 – Algebra III**

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| --- | --- |
|  | **Unit 3:****Algebra III** |
| Unit 3:Algebra III | * Function notation
* **Domain and range of a function**
* Composite functions
* Graphs of functions
* **Graphs of functions with up to three parts to their domains**
 | * Graphs of linear functions
* Finding the equation of a line
* Graphs of quadratic functions
* Inverse functions
* **Graphs of exponential functions**
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**Domain and range of a function**

* **Find the domain of a function. This could be seen as the range of values that can be used as inputs for that function.**
* **Find the range of a function. This could be seen as the range of values that can be outputted from a function.**

**Graphs of functions with up to three parts to their domain**

* **Drawing and sketching of functions; Graphs could be linear, quadratic, exponential and restricted to no more than 3 domains**
* **State the value of *x* for which the equation is not defined**
* **Plot piecewise linear graphs for a series of domains (ranges of x-values)**
* **Plot piecewise linear graphs for real-life situations, e.g. distance-time graphs**

**Graphs of exponential functions (Some appear on GCSE Higher Maths)**

* **Recognise, sketch and interpret graphs of exponential functions *y* = *kx* for positive values of *k* and integer values of *x***
* **Use calculators to explore exponential growth and decay**
* **Set up, solve and interpret the answers in growth and decay problem**
* **Interpretation of graphs; Exponential graphs will be of the form** $y=ab^{x}$ **form** $y=ab^{-x}$**, where** $a$ **and** $b$ **are rational numbers**

Students should already be famialr with and fluent in the following topics. Opportunity to stretch and challenge these topics:

**Function notation, composite and inverse functions**

* Use function notation
* Find f(*x*) + g(*x*) and f(*x*) – g(*x*), 2f(*x*), f(3*x*) etc algebraically
* Know that f –1(*x*) refers to the inverse function
* Find the inverse of a linear function
* For two functions f(*x*) and g(*x*), find gf(*x*)

**Graphs of linear functions**

* Recognise that equations of the form *y* = *mx* + *c* correspond to straight-line graphs in the coordinate plane
* Plot and draw graphs of straight lines of the form *y* = *mx* + *c* with and without a table of values
* Plot and draw graphs of straight lines in the form *ax* + *by* = *c*
* Interpret and analyse information presented in a range of linear graphs:
* find approximate solutions to a linear equation from a graph
* identify direct proportion from a graph

**Graphs of quadratic functions**

* Generate points and plot graphs of simple quadratic functions, then more general quadratic functions
* Identify the line of symmetry of a quadratic graph
* Find approximate solutions to quadratic equations using a graph
* Interpret graphs of quadratic functions from real-life problems
* Identify and interpret roots, intercepts and turning points of quadratic graphs
* Recognise a linear, quadratic, cubic, reciprocal and exponential graph from its shape
* Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function
* Interpret graphs of quadratic functions from real-life problems
* Draw graphs of simple cubic functions using tables of values
* Interpret graphs of simple cubic functions, including finding solutions to cubic equations
* Draw graphs of the reciprocal function  with *x* ≠ 0 using tables of values
* Recognise, sketch and interpret graphs of the reciprocal function  with *x* ≠ 0
* State the value of *x* for which the equation is not defined
* **Students must be comfortable sketching graphs on non-graph paper, indicating critical coordinates such as axis-intercepts.**

# **Unit 4 – Algebra IV**

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|  | **Unit 4:****Algebra IV** |
| Unit 4:Algebra IV | * Quadratic equations
* Simultaneous equations in two unknowns
* **The factor theorem**
* Algebraic proof
* Sequences
 | * Linear inequalities
* Quadratic inequalities
* Indices
* **Limiting value of a sequence**
* **Simultaneous equations in three unknowns**
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**The Factor Theorem**

* **Use of the factor theorem for rational values of the variable for polynomials**
* **Use polynomial division to factorise and solve cubic (and higher order) equations**

**Limiting value of a sequence**

* **Using *n*th terms of sequences, work out the difference between the 16th and 6th terms of the sequence with nth term** $\frac{2n}{n+4}$
* **Find the limiting value of a sequence as *n*→∞**
* **By using the *n*th term, work out the value of any term**
* **Work out the *n*th term of the linear sequence**

**Simultaneous equations in three unknowns**

* **Algebraic solution of linear equations in three unknowns**

Students should already be famialr with and fluent in the following topics. Opportunity to stretch and challenge these topics:

**Quadratic Equations**

* Memorise and use the quadratic formula
* Find the roots of a quadratic function and represent these on a graph
* Set up and solve quadratic equations
* Interpret the solution in the context of the problem
* Solve quadratic equations that need rearranging
* Set up and solve quadratic equations
* Find the roots of a quadratic function algebraically and represent these on a graph
* Find approximate solutions to quadratic equations using a graph

**Simultaneous Equations with two unknowns**

* Write simultaneous equations to represent a situation
* Solve simple simultaneous equations (linear/linear) algebraically eg: where no multiplying or only one variable multiplying required
* Use elimination or substitution to solve simultaneous equations
* Including where both need multiplying
* Interpret the solution in the context of the problem
* Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
* linear / quadratic
* linear / *x*2 + *y*2 = *r*2
* Solve simultaneous equations of circles and straight lines graphically by intersection with exact or approximate answers as necessary
* Set up and solve a pair of simultaneous equations in two variables for each of the above scenarios, including to represent a situation
* Solve simultaneous equations (linear/non-linear) graphically
* Solve simultaneous equations representing a real-life situation, graphically and interpret the solution in the context of the problem
* Find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach

**Algebraic Proof**

* **Answer ‘show that’ questions using consecutive integers (*n*, *n* + 1), squares *a*2, *b*2, even numbers 2*n*, and odd numbers 2*n* +1**
* **Solve ‘Show that’ and proof questions using consecutive integers (*n*, *n* + 1), squares**

**Sequences**

* Find the next term in a sequence, including negative values
* Use function machines to find terms of a sequence
* Write the term-to-term definition of a sequence in words
* Find a specific term in the sequence using position-to-term or term-to-term rules
* Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams
* Recognise such sequences from diagrams and draw the next term in a pattern sequence
* Describe in words a term-to-term sequence and identify which terms cannot be in a sequence
* Use the *n*th term of an arithmetic sequence to generate terms
* Use the *n*th term of an arithmetic sequence to find the first term greater/less than a certain number
* Continue a quadratic sequence and use the *n*th term to generate terms
* Find the *n*th term for a pattern sequence
* Find the *n*th term of a linear/arithmetic sequence
* Find and use (to generate terms) the *n*th term of an arithmetic sequence
* Use the *n*th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number
* Identify which terms cannot be in a sequence by finding the *n*th term
* Solve problems involving sequences from real life situations
* Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences
* Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms
* Continue a Fibonacci sequence, including algebra, negatives, fraction and decimal terms
* Distinguish between arithmetic and geometric sequences
* Find the *n*th term of quadratic sequences
* Distinguish between arithmetic and geometric sequences
* Use finite/infinite and ascending/descending to describe sequences
* Recognise and use simple geometric progressions (*rn* where *n* is an integer, and *r* is a rational number > 0 or a surd
* Continue geometric progression and find term to term rule, including negative, fraction and decimal terms
* Solve problems involving sequences from real life situations

**Linear and Quadratic Inequalities**

* Solve simple linear inequalities in one variable
* Solve two inequalities in *x*, find the solution sets and compare them to see which value of *x* satisfies both
* Solve two linear inequalities in *x*, find the solution sets and compare them to see which value of *x* satisfies both solve linear inequalities in two variables algebraically
* Use the correct notation to show inclusive and exclusive inequalities
* Solve linear inequalities in two variables graphically
* Show the solution set of several inequalities in two variables on a graph
* Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values
* Represent the solution set for inequalities using set notation, i.e. curly brackets and ‘is an element of’ notation
* for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of *x*² – 3*x* – 10 < 0 as {*x*: –3 < *x* < 5}
* Show the solution set of several inequalities in two variables on a graph

**Indices**

* Recall that  = √*n* and  = 3√*n* for any positive number *n*
* Find the value of calculations using indices including positive and negative indices
* Recall that *n*0 = 1 and *n*–1 =  for positive integers n
* Use instances of index laws, including use of zero and negative powers
* Solve problems using index laws
* Use numbers raised to the power zero, including the zero power of 10
* Use instances of index laws, including use of fractional powers
* Find the value of calculations using indices unit fractional indices
* Recall that  = √*n* and  = 3√*n* for any positive number *n*
* Understand that the inverse operation of raising a positive number to a power *n* is raising the result of this operation to the power 
* Solve problems using index laws
* Find the value of calculations using indices including positive, non-unit fractional and negative indices
* Solve problems using index laws
* **Express complicated expressions as a single power of** $x$ **for example:**$\sqrt{x^{\frac{1}{2}}×x^{\frac{7}{2}}}$
* **Solve equations such as:** $x^{-\frac{1}{2}}=3$
* **Solve equations and inequalities such as:** $\sqrt{x}-\frac{10}{\sqrt{x}}=3 x>0$

# **Unit 5 – Coordinate Geometry**

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|  | **Unit 5: Coordinate Geometry** |
| Unit 5:Co-ordinate Geometry | * Parallel and perpendicular lines
* **The distance between two points**
* The midpoint of a line joining two points
* Dividing a line in a given ratio
 | * Equation of a straight line
* The intersection of two lines
* **Equation of a circle (centre (a, b) )**
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**Equation of a Circle (centre (a, b) ) | Centre (0, 0) is Higher GCSE Maths**

* **Understand that** $(x-a)^{2}+(y-b)^{2}=r$**is the equation of a circle with centre (*a, b*) and radius *r***
* **Write down the equation of any circle given centre and radius**
* Draw circles, centre the origin, equation *x*2 + *y*2 = *r*2
* Recognise circle graph from its shape

**Distance between two points**

* **Find the distance between two points using** $l=\sqrt{\left(x\_{2}-x\_{1}\right)^{2}+\left(y\_{2}-y\_{1}\right)^{2}}$
* **Understand this is an extension of Pythagoras’s Theorem**

Students should already be famialr with and fluent in the following topics. Opportunity to stretch and challenge these topics:

**Parallel and Perpendicular Lines**

* Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line
* Select and use the fact that when *y* = *mx* + *c* is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of *m* and a line perpendicular to this line will have a gradient of 
* Explore the gradients of parallel lines and lines perpendicular to each other

**Midpoint of a line segment**

* Find the coordinates of the midpoint of a line segment with a diagram given and coordinates
* Find the coordinates of the midpoint of a line segment from coordinates

**Dividing a line in a given ratio**

* **Use ratio to divide a line into segments.**
* **Use ratio to find the coordinates of a point on a line given the coordinates of two other points**

**Equation of a straight line**

* **Find the equation of the line through two given points**
* **The equation of a straight line *y* = *mx* + *c* and *y* – *y*1 = *m* (*x* – *x*1) and other forms; Including interpretation of the gradient and *y-*intercept from the equation**

**Intersection of two lines**

* **Find where two graphs intersect by solving simultaneous equations through substitution.**

# **Unit 6 – Geometry**

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|  | **Unit 6/7:****Geometry** |
| Unit 6/7:Geometry | * Pythagoras’ theorem
* Angle facts
* Circle theorems
* Geometric proof
* Trigonometry in two dimensions
* Trigonometric functions for angles of any size
* The area of a triangle
* Problems in three dimensions
* Lines and planes in three dimensions
 | * The sine and cosine graphs
* The tangent graph
* **Solution of Trigonometric Equations**
* **Trigonometric identities**

• The sine rule• The cosine rule* Using the sine and cosine rules together
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**Solving Trigonomaetric Equations**

* **Find the solution of simple trigonometric equations in given intervals where equations are to be restricted to single angles**
* **Find the solution of trigonometric equations requiring the use of trigonometric identities (see below) and finding multiple solutions using trigonometric graphs.**
* **Solving quadratic trigonometric equations.**

**Trigonometric Identities**

* **Know and use** $tanθ=\frac{sinθ}{cosθ}$ **and** $sin^{2}θ+cos^{2}θ=1$**, including expressions to be simplified, proofs of identities and equations solved**

Students should already be famialr with and fluent in the following topics. Opportunity to stretch and challenge these topics:

**Pythagoras’ Theorem and Trigonometry**

* Given 3 sides of a triangle, justify if it is right-angled or not
* Calculate the length of the hypotenuse in a right-angled triangle, including decimal lengths and a range of units
* Find the length of a shorter side in a right-angled triangle
* Apply Pythagoras’ Theorem with a triangle drawn on a coordinate grid
* Calculate the length of a line segment AB given pairs of points
* Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question
* Understand the language of planes, and recognise the diagonals of a cuboid
* Solve geometrical problems on coordinate axes
* Understand, recall and use trigonometric relationships and Pythagoras’ Theorem in right-angled triangles, and use these to solve problems in 3D configurations
* Calculate the length of a diagonal of a cuboid
	+ Find the angle between a line and a plane

**Angle Facts**

* Understand ‘regular’ and ‘irregular’ as applied to polygons
* Use the sum of angles of irregular polygons
* Calculate and use the angles of regular polygons
* Use the sum of the interior angles of an *n*-sided polygon
* Use the sum of the exterior angles of any polygon is 360°
* Use the sum of the interior angle and the exterior angle is 180°
* Explain why some polygons fit together and others do not
* Calculate and use the sums of the interior angles of polygons, use the sum of angles in a triangle to deduce and use the angle sum in any polygon and to derive the properties of regular polygons
* Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons
* Calculate the angles of regular polygons and use these to solve problems
* Use angle facts to demonstrate how shapes would ‘fit together’, and work out interior angles of shapes in a pattern
* Identify a line perpendicular to a given line
* Mark perpendicular lines on a diagram and use their properties
* Identify parallel lines
* Mark parallel lines on a diagram and use their properties
* Show step-by-step deduction when solving problems
* Understand and use the angle properties of parallel lines
* Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons

**Circle Theorems and Geometric Proof**

* Identify (name) and draw parts of a circle, including sector, tangent, chord, segment
* Prove and use the facts that:
* the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference
* the angle in a semicircle is a right angle
* the perpendicular from the centre of a circle to a chord bisects the chord
* angles in the same segment are equal
* alternate segment theorem
* opposite angles of a cyclic quadrilateral sum to 180°
* Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point
* Find and give reasons for missing angles on diagrams using:
* circle theorems
* isosceles triangles (radius properties) in circles
* the fact that the angle between a tangent and radius is 90°
* the fact that tangents from an external point are equal in length

**Trigonometric Functions for angles of any size (Graphs of Sine, Cosine and Tangent)**

* Recognise, sketch and interpret graphs of the trigonometric functions (in degrees)
*y* = sin *x*, *y* = cos *x* and *y* = tan *x* for angles of any size
* Know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45° , 60° and 90° and exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60° and find them from graphs
* Use exact values to evaluate calculations such as cos 30° x sin 30°
* Solve equations involving sin, cos or tan

**Sine and Cosine Rule**

* Know the sine and cosine rules, and use to solve 2D problems (including involving bearings)
* Use the sine and cosine rules to solve 3D problems

**Area of a Traingle**

* Know and apply Area = *ab* sin *C* to calculate the area, sides or angles of any triangle
* Calculate the area of a segment

# **Unit 8 – Calculus**

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|  | **Unit 8:****Calculus** |
| Unit 8:Calculus | * **The gradient of a curve**
* **Differentiation**
* **Increasing and decreasing functions**
* **The second derivative**
 | * **Differentiation using standard results**
* **Tangents and normal**
* **Stationary points**
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

* **Know that the gradient function** $\frac{dy}{dx}$ **gives the gradient of the curve and measures the rate of change of y with respect to x**
* **Know that the gradient of a function is the gradient of the tangent at that point.**
* **Differentiation of** $kx^{n}$ **where n is an integer, and the sum of such functions; Including expressions which need to be simplified first, i.e. Given** $y=(3x + 2)(x – 3) $**work out** $\frac{dy}{dx}$**; Given** $y=\frac{5 }{x^{3}}$ **work out** $\frac{dy}{dx}$
* **The equation of a tangent and normal at any point on a curve**
* **Increasing and decreasing functions; When the gradient is positive/negative a function is described as an increasing/decreasing function**
* **Understand and use the notation** $\frac{d^{2}y}{d^{2}x}$ **; Know that** $\frac{d^{2}y}{d^{2}x}$ **measures the rate of change of the gradient function**
* **Use of differentiation to find maxima and minima points (stationary points) on a curve; Determine the nature either by using increasing and decreasing functions or** $\frac{d^{2}y}{d^{2}x}$
* **Using calculus to find maxima and minima in simple problems; i.e.**$ V=49x+\frac{81}{x}$ **for** $x>0$**, Use calculus to show that V has a minimum value and work out the minimum value of V**
* **Sketch/ interpret a curve with known maximum and minimum points**

# **Unit 9 – Matrices**

|  |  |
| --- | --- |
|  | **Unit 9:****Matrices** |
| Unit 9:Matrices | * **Multiplying matrices**
* **Transformations**
* **The identity matrix**
 | * **Transformations of the unit square**
* **Combining transformations**
 |

***Further Maths exclusive content***

By the end of this unit, students should be able to:

**Matrix multiplication**

**Note: All calculations will be restricted to 2 × 2 or 2 × 1 matrices**

* **Multiplying a 2 × 2 matrix by a 2 × 2 matrix or by a 2 × 1 matrix**
* **Multiplication by a scalar**
* **Calculate with the identity matrix I**

**Transformations of matrices**

**Note: All calculations will be restricted to 2 × 2 or 2 × 1 matrices**

* **Transformations of the unit square in the *x* – *y* plane**
* **Transformations restricted to rotations of 90°, 180° or 270° about the origin, reflections in the lines *x* = 0, *y* = 0, *y* = *x, y* = –*x* and enlargements centred on the origin**

**Combinations of transformations using matrices**

**Note: All calculations will be restricted to 2 × 2 or 2 × 1 matrices**

* **Combination of transformations**
* **Using matrix multiplications**
* **Use of i and j notation is not required**